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Design of Supercritical Swept Wings

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Two-dimensional codes for the design and analysis of transonic airfoils are now in common use. Codes have also been developed to analyze the transonic flow past a swept wing in three-dimensional space. In this paper, a computational method to design supercritical swept wings is discussed that is based on a well-posed free boundary problem for the velocity potential. An improved formula for the wave drag is included.

I. The Role of Computational Fluid Dynamics

THERE has been a great deal of interest lately in problems of transonic aerodynamics related to shockless airfoils and supercritical wings. Computational fluid dynamics has turned out to be a very successful tool in this field. For two-dimensional flow, computer codes to design and analyze supercritical wing sections are commonplace. For swept wings in three-dimensional flow, however, many mathematically challenging problems remain open. Some of these problems will be discussed in this paper.

The transonic flow past a swept wing can be described realistically by a velocity potential ϕ. Weak shock waves are represented by adding to the partial differential equation for ϕ artificial viscosity terms that may or may not be in conservation form. Iterative schemes to solve corresponding sets of difference equations numerically are obtained by adding further terms that involve differentiation with respect to an artificial time parameter t. The whole process can be modeled by a partial differential equation for ϕ of the form

$$\Sigma(c^2\delta_{jk}-\phi_{x_j}\phi_{x_k})\phi_{x_jx_k} = -h\max[M^2-1,0]$$
$$\times\Sigma\,|\,\phi_{x_j}\,|\,\phi_{x_k}\phi_{x_jx_jx_k}+\Sigma\alpha_j\phi_{x_jt}+\beta\phi_t$$

where c is the speed of sound determined by Bernoulli's law, $M=|\nabla\phi|/c$ is the Mach number, h is a small positive number related to mesh size, and α_j and β are coefficients specifying the iterative scheme. The normal derivative of ϕ is put equal to zero on the surface of the wing, and the vortex sheet behind is modeled by a linearized boundary condition.

The boundary value problem we have described has been used to develop codes for the calculation of transonic flow past an oblique wing or a swept wing.[1,2] For the implementation, parabolic coordinates x and y are introduced in planes perpendicular to the wing. At each span station z this is accomplished in order to achieve good resolution at the leading and trailing edges. Through a substitution of the form

$$Y=y-f(x,z)$$

a further transformation is made to a rectangular domain

$$-1\le X\le 1,\qquad 0\le Y\le 1,\qquad 0\le Z\le 1$$

that is convenient for the computation. Thus, the surface of the wing $y=f(x,z)$ is mapped onto a region of the plane $Y=0$ with X varying primarily in the direction of the flow and Z indicating the span station. This formulation of the problem is well adapted to equations of design that require changes in the function f.

Meaningful runs of the swept-wing code can be made on a mesh of $152\times10\times12$ points using only 200 time cycles. When this is done, the running time of the code is approximately 1 min on the CRAY computer. Such calculations are relatively easy on a computer of the capacity of the CDC 6600.

II. Entropy Formula for the Wave Drag

The drag has not been calculated very accurately in three dimensions, especially when conservation form of the equation for the velocity potential ϕ has been dropped in order to simulate boundary-layer effects. Examination of computed pressure distributions, however, does seem to give a physically significant measure of the performance of supercritical wings that can be relied on in practice.

An improved formula for the wave drag can be developed from the entropy inequality that is fundamental to the method of artificial viscosity. The idea is that the conservation form of the momentum equation becomes an entropy inequality measuring the drag. The nature of the result is most easily understood in the context of the small-disturbance equation

$$-\tfrac{1}{2}(\phi_x^2)_x+\phi_{yy}=-h(\max[\phi_x,0]\phi_{xx})_x$$

for a potential function ϕ in two dimensions. To capture shocks, an artificial viscosity term $-h(\max[\phi_x,0]\phi_{xx})_x$ has been added to the equation, which expresses the law of conservation of mass. Multiplication by ϕ_x leads to an equation

$$-\tfrac{1}{3}(\phi_x^3)_x+(\phi_x\phi_y)_y-\tfrac{1}{2}(\phi_y^2)_x=-h\phi_x(\max[\phi_x,0]\phi_{xx})_x$$
$$=-h(\max[\phi_x,0]\phi_x\phi_{xx})_x+h\max[\phi_x,0]\phi_{xx}^2$$

describing the conservation of momentum in the direction of x. However, the corresponding artificial viscosity term $-h\phi_x(\max[\phi_x,0]\phi_{xx})_x$ cannot be put in conservation form because, even after an integration by parts, there remains a positive contribution $h\max[\phi_x,0]\phi_{xx}^2$. It is this contribution that leads to the entropy inequality asserting that ϕ_x decreases across a shock.[3]

Integration of the momentum equation reveals that the wave drag is given by the double integral of $h\max[\phi_x,0]\phi_{xx}^2$ over the flow. In regions where the flow is smooth, the integrand is on the order of the mesh width h, so that these regions do not contribute to the wave drag as the mesh is refined. At a normal shock, however, the second derivative ϕ_{xx} becomes singular. Its centered-difference approximation takes the form of a velocity jump across the shock divided by h. The integrand becomes of the order of the cube of the

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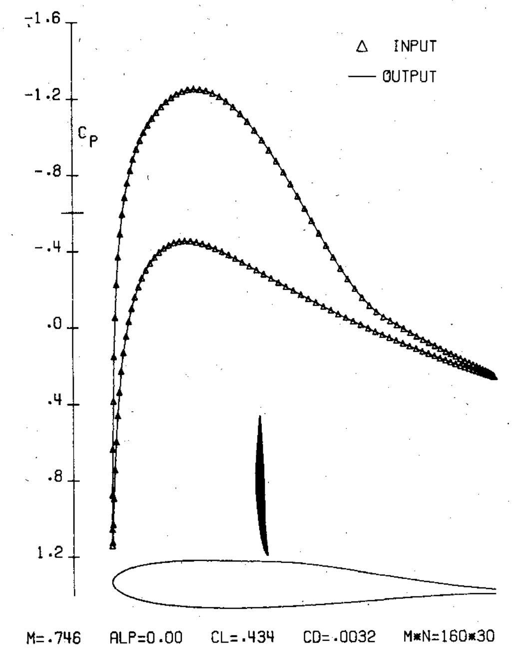

Fig. 1 Shock waves on airfoil modified using artificial viscosity.

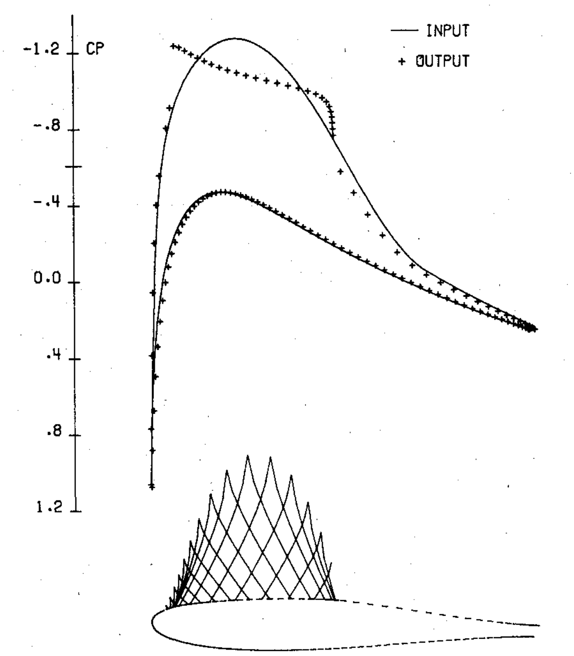

Fig. 2 Mach lines on airfoil modified by the hodograph method.

shock strength, and the double integral tends to a line integral along the shock representing loss of momentum. A familiar application of the divergence theorem also shows that the double integral reduces to a standard line integral for the wave drag.

For the more general swept-wing code, the analogous expression for the wave drag coefficient C_{DW} becomes

$$C_{DW} = \Sigma A h^4 \max [M^2 - 1, 0] \phi_{ss}^2$$

where the summation is extended over all mesh points, s is a rotated coordinate measured in the direction of the flow, and A is a factor depending on the coordinate system and the method of differencing. This representation shows in detail how the drag is distributed along the shocks.

We have incorporated the entropy formula for the wave drag in a two-dimensional code for the analysis of transonic flow over airfoils. Runs have been made to compare the code with experiment. For the Whitcomb wing,[4] the new formula provides good agreement not only near design, but also in an off-design case at very high lift. Experimental and theoretical drag coefficients agree to within 0.001 in the four cases that were published. The quantity $A\max[M^2 - 1, 0]\phi_{ss}^2$ also serves to give an instructive plot of shock locations by means of arcs that specify what the corresponding drag is (see Fig. 1).

In the case of the swept-wing code, the entropy formula leads to a decisive improvement in the calculation of the wave drag. Previously, this could not be distinguished from the much more elusive induced drag due to lift, because all that was computed was an integral of the pressure over the wing. The old formula gave results for the wave drag that were invariably too large because dropping the precise form of the law of conservation of mass introduces errors of second order in the shock strength. The new formula is visibly of third order in the shock strength, as it should be, and it agrees with the two-dimensional calculations that have been correlated with experiment. It is accurate enough for use in combination with an optimization routine to design swept wings with minimum wave drag. The resolution can be further enhanced by linear extrapolation to zero mesh size.

III. Transonic Free Boundary Problem

We are now in a position to attack the problem of designing swept wings with prescribed pressure distributions. At transonic speeds, it is onerous to ask for shockless flow in three dimensions.[5,6] However, we shall discuss a method of artificial viscosity that does seem to offer some prospect of improving the design and reducing the drag of swept wings in various three-dimensional configurations. The scheme provides an alternative to outright minimization of the drag, but it can also be coupled to such an optimization with respect to the prescribed pressure.[7] The method is based on a free boundary problem in which the speed is prescribed continuously over only a portion of the wing. The free boundary problem seems to be well posed provided that shocks satisfying the entropy inequality are allowed to appear in the interior of the flow.

Two-dimensional codes have been written to design an airfoil with a given pressure distribution. For transonic flow there are two possible approaches to this problem. One of them is to introduce complex characteristic coordinates in the hodograph plane in order to obtain a shockless airfoil whose pressure distribution only deviates from the prescribed data in the supersonic zone.[8] The other is to add artificial viscosity in the physical plane so that shocks are smeared out while the data are fitted perfectly over the whole profile.[9]

The hodograph method has the advantage that it determines supercritical wing sections with low levels of wave drag automatically. The method of artificial viscosity is relatively easy to implement as a robust code, but it allows shocks with significant wave drag to remain inside the flow even when the pressure at the profile is smooth. To apply the design codes, one must exercise some skill in choosing the pressure distribution that is assigned. The theory of shockless airfoils can be helpful in making an initial choice (compare Figs. 1 and 2). To eliminate shocks, it is desirable to use a pressure distribution that is peaky near the leading edge of the wing as well as smooth at the rear of the supersonic zone.

Because the hodograph method fails in three-dimensional space, we rely on artificial viscosity to develop a code for the design of supercritical swept wings. Our work is based on Refs. 1 and 2.

To avoid questions of closure and other complications with the geometry, we formulate a free boundary problem in which the pressure p, or rather the speed $q = |\nabla \phi|$, is assigned over only a limited part of the surface of the wing. Let $y = f(x,z)$ be the equation of that surface, let ϕ stand for the velocity

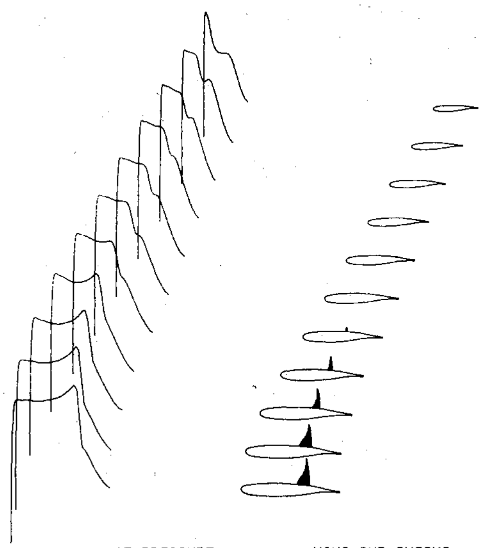

Fig. 3 Pressure on swept wing showing shock near the wall.

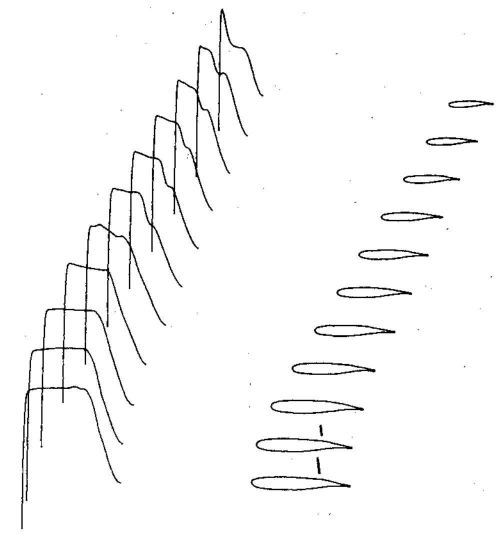

Fig. 4 Smoothed pressure on swept wing with shocks suppressed.

potential of a corresponding flow with given speed at infinity, let $y = f_0(x,z)$ represent a surface that is supposed to lie inside the eventual wing, and let $q_0(x,z)$ specify a prescribed distribution of speed. For the design problem we ask that $f(x,z) \geq f_0(x,z)$ everywhere, assuming that $y > f(x,z)$ defines the region of flow. In addition, we ask that the free boundary condition

$$Q(f,f_x,f_z) = q_0(x,z)^2 - q(x,z)^2 = 0$$

be fulfilled wherever the strict form $f(x,z) > f_0(x,z)$ of the inequality holds. It will be seen that these requirements define a wing on which the prescribed values q_0 of the speed are undertaken only at points where they are sufficiently big. The result is a patch of wing surface on which unwanted shocks may be smoothed out.

The free boundary problem that we have described can be solved numerically by an iterative scheme generalizing one that has been applied successfully to analogous questions arising in computational magnetohydrodynamics.[10] Let both ϕ and f depend on the artificial time parameter t, and suppose that ϕ satisfies the partial differential equation formulated above for transonic flow. We allow the surface $y = f(x,z,t)$ to vary with t according to the following rules. For all t we require that $f \geq f_0$. However, when $f > f_0$, we put

$$a_1(\phi_x f_{xt} + \phi_z f_{zt}) + f_t = Q + a_2(Q_{f_x} Q_x + Q_{f_z} Q_z)$$

with the coefficients a_j chosen to make the scheme converge. This partial differential equation for f is approximated by finite difference equations determining the free surface in the limit as $t \to \infty$. The extra terms on the right are suggested by the Lax-Wendroff scheme, whereas those on the left are motivated by linearized flow theory.[11] For the most part, second-order accurate central differences are used, but the approximations of f_{xt}, f_{zt}, Q_x, and Q_z should be first order and should be retarded according to the method of characteristics so that stability of the iterative scheme is achieved. To solve the difference equations for f, we march in a direction opposed to the flow.

A code that implements our design concept has been written. Computational experience with convergence of the method is encouraging. More specifically, the addition of the term in f_{xt} facilitates convergence when the flow is transonic. Our success is perhaps surprising in view of the fact that flux is stationary at sonic speed. The rate of convergence for the new design code may be enhanced by the simple device of fixing the section lift coefficient at each span station while varying the corresponding angles of attack and twist.

The design code is an engineering tool of practical significance. It can be applied, in a fashion suggested by wind-tunnel techniques, to add material to an underlying wing structure in order to obtain supercritical flow with only weak shocks of negligible wave drag. As a test case, we have attempted to suppress the shocks at the root section of a swept wing that was constructed from shockless airfoils. Figures 3 and 4 show the pressure distribution and shock waves on the upper surface of the wing before and after application of the design code. The reduction in wave drag is gratifying, as is indicated by the figures, where shocks are plotted using arcs whose lengths record the associated drag. Engine nacelles or a possible fuselage can be modeled in the computation by imposing a linearized boundary condition at the wall.

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